

Pure gauge configurations and tachyon solutions to string field theories equations of motion

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Pure gauge configurations and tachyon solutions to string field theories equations of motion

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ABSTRACT: In construction of analytical solutions to open string field theories pure gauge configurations parameterized by wedge states play an essential role. These pure gauge configurations are constructed as perturbation expansions and to guaranty that these configurations are asymptotical solutions to equations of motion one needs to study convergence of the perturbation expansions. We demonstrate that for the large parameter of the perturbation expansion these pure gauge truncated configurations give divergent contributions to the equation of motion on the subspace of the wedge states. We perform this demonstration numerically for the pure gauge configurations related to tachyon solutions for the bosonic and NS fermionic SFT. By the numerical calculations we also show that the perturbation expansions are cured by adding extra terms. These terms are nothing but the terms necessary to make valued the Sen conjectures.

KEYWORDS: Tachyon Condensation, D-branes, String Field Theory

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1 Introduction

Finding nontrivial analytic solutions to classical string field theory (SFT) is one of the long-standing problems in string theory. The first nontrivial solution of the SFT equation of motion has been found by Schnabl [1] for the Witten open bosonic SFT [2]. The Schnabl

paper has attracted a lot of attention [3]- [26]. It turns out that the tachyon solution is closely related to pure gauge solutions. More precisely, Schnabl's solution is a regularization of a singular limit of a special pure gauge configuration [1, 3]. The existence of pure gauge solutions to the bosonic SFT equation of motion is provided by the Chern-Simons form of the Witten cubic action. The Schnabl solution is distinguished by its relation to a true vacuum of the SFT, i.e. the vacuum on which the Sen conjectures [35] are realized.

Schnabl's result has been generalized to the cubic super SFT (SSFT)¹ [36, 37] by Erler [29]. It is natural to expect existence of a pure gauge solution to the cubic super SFT (SSFT) equation of motion. However there is no reason for the superstring case to deal with the Sen conjecture, since the perturbative vacuum is stable (there is no tachyon). However a nontrivial (not pure gauge) solution to the SSFT equation of motion does exist [29].²

To deal with the tachyon condensation for the fermionic string one has to incorporate the $GSO(-)$ sector.³ A solution to the equation of motion of the cubic SFT describing the NS string with both the $GSO(+)$ and $GSO(-)$ sectors has been constructed in [30] (the AGM solution for short). For this solution the first Sen conjecture has been checked analytically [30]. A solution to the equation of motion for the non-polynomial SSFT [44] has been obtained in [27, 28]. This construction became clear after a realization of an explicit relation between the pure gauge solutions for the cubic superstring field theories and non-polynomial ones found by Fuchs and Kroyter [31, 32] (see also a recent discussion in [33]).

In matrix notations [42] the equation of motion for the NS fermionic SFT has the form

$$\widehat{Q}\widehat{\Phi} + \widehat{\Phi} \star \widehat{\Phi} = 0, \tag{1.1}$$

that is the same as the equation of motion for the open bosonic and cubic superstring SFTs (one has just removed the hats in the later cases). $\widehat{\Phi}$ is 2×2 matrix where the components are string fields belonging to the $GSO(+)$ and $GSO(-)$ sectors. Pure gauge solutions can be written as

$$\widehat{\Phi} = -\widehat{Q}\widehat{\Omega} \star \widehat{\Omega}^{-1}, \tag{1.2}$$

where the parity of the entries of $\widehat{\Omega}$ must be adjusted [30]. For solution of Schnabl's form the following formula is relevant

$$\widehat{\Omega} = \widehat{1} - \lambda\widehat{\varphi}, \tag{1.3}$$

where $\widehat{\varphi}$ are special string fields defining the choice of a solution.

In (1.2) $\widehat{\Omega}^{-1}$ is understood as a geometric series that gives

$$\widehat{\Phi}(\lambda) = \sum_{n=0}^{\infty} \lambda^{n+1} \widehat{Q}\widehat{\varphi} \star \widehat{\varphi}^n. \tag{1.4}$$

¹We call the NS fermionic SFT the NS string field theory with two sectors, $GSO(+)$ and $GSO(-)$ and we call the superstring SFT (SSFT) the NS string field theory with the $GSO(+)$ sector only.

²The physical meaning of this solution is still unclear for us. It may happen that it is related with a spontaneous supersymmetry breaking (compare with [38]).

³Let us remind that the NS fermionic SFT with two sectors is used to describe non-BPS branes. The Sen conjecture has been checked by level truncations for the non-polynomial and cubic cases in [46] and [40], respectively.

For a special $\widehat{\varphi}$ the pure gauge solution can be cast into the form

$$\widehat{\Phi}(\lambda) = \sum_{n=0}^{\infty} \lambda^{n+1} \widehat{\phi}'_n, \tag{1.5}$$

where the states $\widehat{\phi}'_n$ are defined for any real $n \geq 0$ and are made of the wedge state [47–49] (see explicit formula in section 2, equations (2.16), (2.20)–(2.27)).

The Schnabl solution for the bosonic SFT as well as the Erler and AGM solutions for the fermionic SFTs consist of two pieces. The first piece is $\widehat{\Phi}(1)$. The second piece consists of the so called phantom terms in the terminology of [31]. The true solution to the equation of motion is defined by the limit:

$$\widehat{\Phi}^R = \lim_{N \rightarrow \infty} \left[\sum_{n=0}^N \widehat{\phi}'_n - \widehat{\psi}_N \right]. \tag{1.6}$$

The phantom term for the Schnabl solution has appeared as an intrinsic part of the Schnabl construction [1].⁴ In the case of the Erler and AGM solutions phantom terms are added [29, 30] to satisfy the equation of motion contracted with solutions themselves and to provide the Sen conjecture.

There is also another argument to add an extra term to the pure gauge configurations. It is a convergence argument, or in other words, a requirement that a perturbative solution must be also an asymptotical weak solution on a subspace. As a subspace it is reasonable to consider a subspace spanned by the wedge states. As it was observed in [33] on the example of the cubic open SSFT, the perturbatively defined pure gauge configuration, related to the Erler solution, fails to be a solution to the equation of motion when contracted with wedge states. It has been shown that it is possible to cure the perturbation expansion by adding extra phantom terms that are just the terms that have been used previously to provide that the equation of motion contracted with the solution itself be satisfied [29]. Similar numerical results for the bosonic string have been reported in [34].

The main purpose of this paper is to study the convergence of the pure gauge configurations related to the tachyon solution [30] and to test the corresponding phantom terms.

We show that

- the pure gauge solution related to the AGM solution and defined perturbatively,

$$\widehat{\Phi}_N(\lambda) = \sum_{n=0}^N \lambda^{n+1} \widehat{\phi}'_n, \tag{1.7}$$

is divergent at $\lambda = 1$ in the sense that the correlator

$$\langle\langle \widehat{\phi}_m, \widehat{Q}\widehat{\Phi}_N(1) + \widehat{\Phi}_N(1) \star \widehat{\Phi}_N(1) \rangle\rangle, \tag{1.8}$$

⁴In [3, 4] it has been checked that the phantom term for the bosonic tachyon solution provides the equation of motion contracted with the solution itself.

does not go to zero for any fixed m and $N \rightarrow \infty$ and

$$\langle\langle \hat{\phi}_m, \hat{Q}\hat{\Phi}(\lambda) + \hat{\Phi}(\lambda) \star \hat{\Phi}(\lambda) \rangle\rangle \neq 0 \text{ for } |\lambda| \geq 1, \quad (1.9)$$

meanwhile

$$\langle\langle \hat{\phi}_m, \hat{Q}\hat{\Phi}(\lambda) + \hat{\Phi}(\lambda) \star \hat{\Phi}(\lambda) \rangle\rangle = 0 \text{ for } |\lambda| < 1 \quad (1.10)$$

- it is possible to cure the perturbation expansion $\hat{\Phi}_N(1)$ by adding extra terms $\hat{\psi}_N$,

$$\hat{\Phi}_N(1) \rightarrow \hat{\Phi}_N^R(1) \equiv \hat{\Phi}_N(1) + \hat{\psi}_N, \quad (1.11)$$

so that the correlator

$$\langle\langle \hat{\phi}_m, \hat{Q}\hat{\Phi}_N^R(1) + \hat{\Phi}_N^R(1) \star \hat{\Phi}_N^R(1) \rangle\rangle, \quad (1.12)$$

goes to zero when $N \rightarrow \infty$.

- $\hat{\psi}_N$ is just the same term that has been used previously to provide that the equation of motion contracted with the solution itself be satisfied [30].

The paper is organized as follows.

In section 2 a matrix formulation for the NS fermionic SFT is recalled and perturbative parameterizations of special pure gauge configurations are presented. These pure gauge configurations are used in the tachyon fermionic solution [30].

Section 3 is devoted to the pure $GSO(+)$ sector and we give an explicit demonstration that $\lambda = 1$ limit of the pure gauge configurations used in the Erler construction is in fact a singular point and that it is possible to use a simple prescription to cure divergences and this prescription gives the same answer as the requirement of validity of the equation of motion contracted with the solution itself.

Section 4 is devoted to the NS fermionic string including the $GSO(-)$ sector. In this case we do not have simple formulae for correlators as we do have for the Erler solution. To demonstrate that $\lambda = 1$ limit of the pure gauge configurations is in fact a singular point we use numerical calculations. We also use numerical calculation to find phantom terms that cure divergences. We show that the found phantom terms are the same as found before from the requirement of validity of the equation of motion contracted with the solution itself.

In section 5 for completeness we collect the similar calculations for the Schnabl solution to the bosonic SFT equation of motion adjusting the presentation of materials to our discussion of the fermionic strings in section 3 and section 4.

In appendix we collect correlators of wedge states with insertions used in the construction of the solution to equation of motion.

2 Set up

2.1 Perturbative pure gauge solution

2.1.1 NS fermion string including $GSO(-)$ in matrix notations

We begin with the action [40] (the ABKM action for short) in matrix notations [42]

$$S[\hat{\Phi}] = \frac{1}{2} \langle \hat{Y}_{-2} \hat{\Phi}, \hat{Q}\hat{\Phi} \rangle + \frac{1}{3} \langle \hat{Y}_{-2} \hat{\Phi}, \hat{\Phi} \star \hat{\Phi} \rangle, \quad (2.1)$$

the string field $\widehat{\Phi}$ is given by

$$\widehat{\Phi} = \Phi_+ \otimes \sigma_3 + \Phi_- \otimes i\sigma_2, \quad (2.2)$$

where Φ_+, Φ_- are string fields which belong to $GSO(\pm)$ [39] sectors respectively, and

$$\widehat{Q} = Q \otimes \sigma_3, \quad \widehat{Y}_{-2} = Y_{-2} \otimes \sigma_3, \quad (2.3)$$

σ_i are Pauli matrices, Q is the BRST charge and Y_{-2} is a double step picture changing operator [41].

The parity assignment and σ_i algebra lead to the Leibnitz rule

$$\widehat{Q}(\widehat{\Phi} \star \widehat{\Psi}) = (\widehat{Q}\widehat{\Phi}) \star \widehat{\Psi} + (-)^{|\widehat{\Phi}|} \widehat{\Phi} \star (\widehat{Q}\widehat{\Psi}), \quad (2.4)$$

where

$$|\widehat{\Phi}| \equiv |\Phi_+|. \quad (2.5)$$

The equation of motion in the matrix notations reads

$$\widehat{Q}\widehat{\Phi} + \widehat{\Phi} \star \widehat{\Phi} = 0. \quad (2.6)$$

If $\widehat{\Phi}$ is a nontrivial solution for (2.6), then it has to be Grassman odd (i.e. $|\widehat{\Phi}| = 1$). A pure gauge solution to (2.6) is

$$\widehat{\Phi} = -\widehat{Q}\widehat{\Omega} \star \widehat{\Omega}^{-1}. \quad (2.7)$$

$\widehat{\Omega}$ is even, (i.e. $|\widehat{\Omega}| = 0$) and in components has the form:

$$\widehat{\Omega} = \Omega_+ \otimes I + \Omega_- \otimes \sigma_1. \quad (2.8)$$

We parameterize $\widehat{\Omega}$ as [30]

$$\widehat{\Omega} = \widehat{1} - \lambda\widehat{\varphi}, \quad (2.9)$$

where

$$\widehat{\varphi} = \varphi_+ \otimes I + \varphi_- \otimes \sigma_1, \quad (2.10)$$

φ_+ and φ_- are components of the gauge field $\widehat{\varphi}$ and they belong to the $GSO(+)$ and $GSO(-)$ sectors respectively. The Grassman parities of φ_+ and φ_- are opposite.

For this $\widehat{\Omega}$ the pure gauge configuration (2.7) has the form

$$\widehat{\Phi}(\lambda) = \lambda\widehat{Q}\widehat{\varphi} \star \frac{1}{1 - \lambda\widehat{\varphi}}. \quad (2.11)$$

One can expand the expression (2.11) in λ to get

$$\widehat{\Phi}(\lambda) = \sum_{n=0}^{\infty} \lambda^{n+1} \widehat{Q}\widehat{\varphi} \star \widehat{\varphi}^n. \quad (2.12)$$

In [30] has been proposed to take the following form for φ_+ and φ_-

$$\varphi_+ = FBcF, \quad (2.13)$$

$$\varphi_- = FB\gamma F. \quad (2.14)$$

Here φ_+ and φ_- are written in the split-string notations. The detailed relation of the split-string formalism and CFT has been elaborated by Okawa and Erler [3, 19] for bosonic SFT and by Erler [27, 29] for SSFT. We sketch the formulae related the split-string formalism and CFT in appendix.

The explicit expansion of the gauge configuration (2.11) in the parameter λ is

$$\widehat{\Phi}(\lambda) = \sum_{n=0}^{\infty} \lambda^{n+1} \widehat{\phi}'_n, \quad (2.15)$$

where

$$\widehat{\phi}'_n = \zeta'_n \otimes \sigma_3 + \xi'_n \otimes i\sigma_2. \quad (2.16)$$

In components

$$\widehat{\Phi}(\lambda) = \Phi_+(\lambda) \otimes \sigma_3 + \Phi_-(\lambda) \otimes i\sigma_2, \quad (2.17)$$

where

$$\Phi_+(\lambda) = \sum_{n=0}^{\infty} \lambda^{n+1} \zeta'_n, \quad (2.18)$$

$$\Phi_-(\lambda) = \sum_{n=0}^{\infty} \lambda^{n+1} \xi'_n. \quad (2.19)$$

In the split-string notations we have [30]

$$\zeta'_0 = FcK BcF + FB\gamma^2 F, \quad (2.20)$$

$$\xi'_0 = FcKB\gamma F + \frac{1}{2}FB\gamma KcF + \frac{1}{2}FB\gamma cKF, \quad (2.21)$$

$$\zeta'_n = \psi'_n + \chi'_n, \quad n > 0, \quad (2.22)$$

$$\xi'_n = \vartheta'_n + \eta'_n, \quad n > 0, \quad (2.23)$$

where

$$\psi'_n = Fc\Omega^n K BcF, \quad n > 0, \quad (2.24)$$

$$\chi'_n = F\gamma\Omega^n K B\gamma F, \quad n > 0, \quad (2.25)$$

$$\vartheta'_n = F\gamma\Omega^n K BcF, \quad n > 0, \quad (2.26)$$

$$\eta'_n = Fc\Omega^n K B\gamma F, \quad n > 0. \quad (2.27)$$

2.1.2 NS superstring

To recover the Erler form [29] of the pure gauge configuration $\Phi_S(\lambda)$ from (2.7)–(2.18) one has to take $\varphi_- = 0$ in (2.10) and φ_+ as in (2.13). This gives

$$\Phi_S(\lambda) = \sum_{n=0}^{\infty} \lambda^{n+1} \psi'_n, \quad (2.28)$$

where

$$\psi'_0 = FcK BcF + FB\gamma^2 F, \quad (2.29)$$

$$\psi'_n = Fc\Omega^n K BcF, \quad n > 0. \quad (2.30)$$

Therefore, the first term in the r.h.s. of (2.22) coincides with the Erler term (2.30) and the term (2.20) coincides with (2.29).

2.1.3 Bosonic string

Comparing our results with the case of the bosonic SFT we use the Schnabl form of the pure gauge configuration

$$\Phi_B(\lambda) = \sum_{n=0}^{\infty} \lambda^{n+1} \varphi'_n, \quad (2.31)$$

where

$$\varphi'_0 = FcKBcF, \quad (2.32)$$

$$\varphi'_n = Fc\Omega^n KBcF, \quad n > 0. \quad (2.33)$$

2.2 Asymptotic and weak asymptotic solutions

By construction the pure gauge configuration (2.11) solves equation of motion (2.6) at each order in λ . However this does not mean that the constructed pure gauge configuration has a meaning within a non-perturbative framework. In particular, one can wonder if the gauge configuration defines an asymptotic solution to the equation of motion. Let us remind the definition of asymptotic solution to the SFT equation of motion. $\widehat{\Phi}_N$ is called an asymptotic solution to (2.6), if

$$\lim_{N \rightarrow \infty} \left(\widehat{Q}\widehat{\Phi}_N + \widehat{\Phi}_N \star \widehat{\Phi}_N \right) = 0. \quad (2.34)$$

In our cases we can deal only with weak asymptotic solutions. Let us remind $\widehat{\Phi}_N$ is called the weak asymptotic solution on a subspace \mathcal{S} if

$$\lim_{N \rightarrow \infty} \langle \langle \psi, \widehat{Q}\widehat{\Phi}_N + \widehat{\Phi}_N \star \widehat{\Phi}_N \rangle \rangle = 0 \quad (2.35)$$

for any $\psi \in \mathcal{S}$.

By construction we can guaranty that the pure gauge configuration is a perturbative solution in a sense

$$\widehat{Q}\widehat{\phi}'_n + \sum_{m=0}^n \widehat{\phi}'_m \star \widehat{\phi}'_{n-m} = 0, \quad (2.36)$$

but we cannot a priori guaranty that (2.35) takes place.

2.2.1 Notations for correlators

We are going to consider the validity of equation of motion on a subspace spanned by wedge state $\psi_n, \chi_n, \eta_n, \vartheta_n$ with $n > 0$ and ζ'_0, ξ'_0 . We use the following notations:

- for the NS fermion string case (the AGM pure gauge configuration)

$$\mathcal{R}_+(\text{field}|N, \lambda) \equiv \langle \langle \text{field}, Q\Phi_{+,N}(\lambda) + \Phi_{+,N}(\lambda) \star \Phi_{+,N}(\lambda) - \Phi_{-,N}(\lambda) \star \Phi_{-,N}(\lambda) \rangle \rangle, \quad (2.37)$$

$$\mathcal{R}_-(\text{field}|N, \lambda) \equiv \langle \langle \text{field}, Q\Phi_{-,N}(\lambda) + \Phi_{+,N}(\lambda) \star \Phi_{-,N}(\lambda) - \Phi_{-,N}(\lambda) \star \Phi_{+,N}(\lambda) \rangle \rangle, \quad (2.38)$$

where $\Phi_{\pm,N}(\lambda)$ are defined according to (2.18), (2.19) by

$$\begin{aligned} \widehat{\Phi}_{+,N}(\lambda) &= \sum_{n=0}^N \lambda^{n+1} \zeta'_n, \\ \widehat{\Phi}_{-,N}(\lambda) &= \sum_{n=0}^N \lambda^{n+1} \xi'_n; \end{aligned} \quad (2.39)$$

- for superstring case (the Erler pure gauge configuration)

$$\mathcal{R}_S(\text{field}|N, \lambda) \equiv \langle\langle \text{field}, Q\Phi_{S,N}(\lambda) + \Phi_{S,N}(\lambda) \star \Phi_{S,N}(\lambda) \rangle\rangle, \quad (2.40)$$

where $\Phi_{S,N}(\lambda)$ is given defined according to (2.28) by

$$\Phi_{S,N}(\lambda) = \sum_{n=0}^N \lambda^{n+1} \psi'_n; \quad (2.41)$$

- for the bosonic string (the Schnabl pure gauge configuration)

$$\mathcal{R}_B(\text{field}|N, \lambda) \equiv \langle \text{field}, Q\Phi_{B,N}(\lambda) + \Phi_{B,N}(\lambda) \star \Phi_{B,N}(\lambda) \rangle, \quad (2.42)$$

where $\Phi_{B,N}(\lambda)$ is given by

$$\Phi_{B,N}(\lambda) = \sum_{n=0}^N \lambda^{n+1} \varphi'_n. \quad (2.43)$$

3 Pure gauge configurations and the Erler solution

3.1 Pure gauge configurations

Let us check validity of the equation of motion in weak sense on the states ψ_K . For this purpose we use the following correlators⁵

$$\begin{aligned} \langle\langle \psi_K, Q\Phi_{S,N} \rangle\rangle &= \frac{\lambda^2}{\pi^2} \frac{1 - \lambda^N}{1 - \lambda}, & K > 0, \\ \langle\langle \psi_K, \Phi_{S,N} \star \Phi_{S,N} \rangle\rangle &= -\frac{\lambda^2}{\pi^2} \frac{1 - \lambda^{N+1}}{1 - \lambda}, & K > 0, \\ \langle\langle \psi'_0, Q\Phi_{S,N} \rangle\rangle &= 0, \\ \langle\langle \psi'_0, \Phi_{S,N} \star \Phi_{S,N} \rangle\rangle &= 0, \end{aligned} \quad (3.1)$$

to get

$$\begin{aligned} \mathcal{R}_S(\psi_K|N, \lambda) &= -\frac{\lambda^{N+2}}{\pi^2}, & K > 0, \\ \mathcal{R}_S(\psi'_0|N, \lambda) &= 0. \end{aligned} \quad (3.2)$$

Taking the limit $N \rightarrow \infty$ for $\lambda < 1$ we have for an arbitrary $K > 0$

$$\mathcal{R}_S(\psi_K|\infty, \lambda) = 0. \quad (3.3)$$

Note that for ψ'_K

$$\mathcal{R}_S(\psi'_K|N, \lambda) = \frac{d}{dK} \mathcal{R}_S(\psi_K|N, \lambda) = 0, \quad K > 0, \quad (3.4)$$

in other words for $\lambda < 1$ the field $\Phi_{S,\infty}(\lambda)$ solves the equation of motion when contracted with states from the subspace spanned by ψ_K, ψ'_K, ψ'_0 . This fact is natural for the solution obtained by the iteration procedure (see section 4).

⁵From here $\langle\langle \dots \rangle\rangle = \langle Y_{-2} \dots \rangle$.

From equation (3.2) one sees that for $\lambda = 1$ the string field $\Phi_{S,N}(1)$ does not solve the equation of motion even in weak sense

$$\mathcal{R}_S(\psi_K|N, 1) = -\frac{1}{\pi^2}. \quad (3.5)$$

We have to note that using (3.2) and (3.4) we obtain that action on the pure gauge configuration equals zero

$$S(\Psi_{S,N}(\lambda)) \equiv 0. \quad (3.6)$$

3.2 The Erler solution

Let us add to $\Phi_{S,N}(1) \equiv \sum_{n=0}^N \psi'_n$ two extra terms

$$\Phi_{S,N}^R(a, b) = \Phi_{S,N}(1) + a\psi_N + b\psi'_N, \quad (3.7)$$

and find a and b from the requirement of validity of the equation of motion in weak sense

$$\begin{aligned} \mathcal{R}_S^R(\psi_K|N, 1, a, b) &= 0, & K > 0, \\ \mathcal{R}_S^R(\psi'_0|N, 1, a, b) &= 0. \end{aligned} \quad (3.8)$$

Here by the superscript R we denote that we contract some field with equations of motion for configuration with added phantom terms:

$$\mathcal{R}_S^R(\text{field}|N, 1, a, b) \equiv \langle\langle \text{field}, Q\Phi_{S,N}^R + \Phi_{S,N}^R \star \Phi_{S,N}^R \rangle\rangle. \quad (3.9)$$

Simple calculations based on correlators [29] show that $a = -1$ and $b = -\frac{1}{2}$. Indeed,

$$\begin{aligned} \langle\langle \psi_K, Q\Phi_{S,N}^R(a, b) \rangle\rangle &= \frac{(1+a)N + aK + 2a + b}{\pi^2}, & K > 0, \\ \langle\langle \psi_K, \Phi_{S,N}^R(a, b) \star \Phi_{S,N}^R(a, b) \rangle\rangle &= -\frac{(1+a)N + aK + 3a + b + 1}{\pi^2}, & K > 0, \\ \langle\langle \psi'_0, Q\Phi_{S,N}^R(a, b) \rangle\rangle &= 0, \\ \langle\langle \psi'_0, \Phi_{S,N}^R(a, b) \star \Phi_{S,N}^R(a, b) \rangle\rangle &= -\frac{a(N + 2 + a(N + \frac{3}{2}) + b)}{\pi^2} \end{aligned} \quad (3.10)$$

and we see that

$$\begin{aligned} \mathcal{R}_S^R(\psi_K|N, 1, a, b) &= -\frac{1+a}{\pi^2}, & K > 0, \\ \mathcal{R}_S^R(\psi'_0|N, 1, a, b) &= -\frac{a(N + 2 + a(N + \frac{3}{2}) + b)}{\pi^2} \end{aligned} \quad (3.11)$$

are equal to zero only for $a = -1$, $b = -\frac{1}{2}$.

Next we calculate following correlators

$$\langle\langle \Phi_{S,N}^R, Q\Phi_{S,N}^R \rangle\rangle = \frac{2a}{\pi^2} \left[(1+a)N + a + b \right], \quad (3.12)$$

$$\langle\langle \Phi_{S,N}^R, \Phi_{S,N}^R \star \Phi_{S,N}^R \rangle\rangle = -\frac{3a}{\pi^2} \left[(1+a)N + \frac{3}{2}a + b + 1 \right]. \quad (3.13)$$

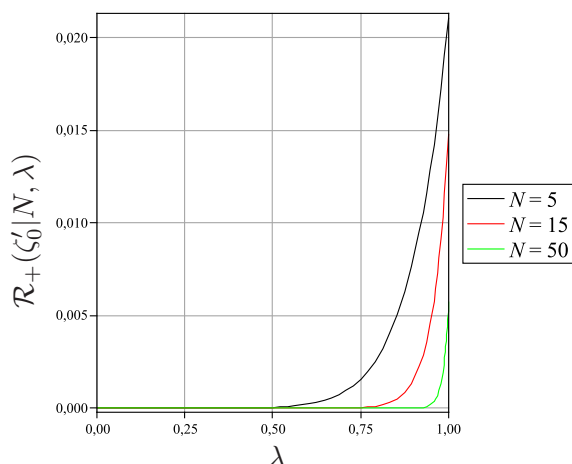


Figure 1. Contraction of equations of motion for $\Phi_{+,N}(\lambda)$, $N = 5, 10, 15, 20, 50$ with ζ'_0 .

Using these expressions we obtain contraction of the solution with the equation of motion

$$\langle\langle \Phi_{S,N}^R, Q\Phi_{S,N}^R + \Phi_{S,N}^R \star \Phi_{S,N}^R \rangle\rangle = -\frac{a}{\pi^2} \left[(1+a)N + \frac{5}{2}a + b + 3 \right] \quad (3.14)$$

equals zero when $a = -1$ and $b = -\frac{1}{2}$. Following Erler [29] we can calculate action on this solution for arbitrary a and b :

$$S = \frac{1}{2} \langle\langle \Phi_{S,N}^R, Q\Phi_{S,N}^R \rangle\rangle + \frac{1}{3} \langle\langle \Phi_{S,N}^R, \Phi_{S,N}^R \star \Phi_{S,N}^R \rangle\rangle = -\frac{a(2+a)}{2\pi^2}. \quad (3.15)$$

If we take $a = -1$ we get

$$S = \frac{1}{2\pi^2}. \quad (3.16)$$

It's interesting to note that b does not contribute to the value of action.

4 Pure gauge configurations and the AGM solution

4.1 Pure gauge configuration as asymptotic solution for equation of motion

Let us consider perturbative solution (2.39) and check whether it solves equations of motion in weak sense on the subspace spanned by $\psi_n, \chi_n, \vartheta_n, \eta_n$ for $n \geq 1$ and ζ'_0, ξ'_0 by direct computations. For this purpose we evaluate expressions (2.37) and (2.38) for the mentioned fields. In figure 1 we see the dependence of $\mathcal{R}_+(\zeta'_0|N, \lambda)$ on λ (recall that N is the number at which we truncate the solution). For the large N the curve is closer to the λ -axis when $\lambda < 1$. This means that this perturbative pure gauge configuration does in fact solve the equation of motion in the weak sense.

In figure 2 we see $\mathcal{R}_+(\psi_K|N, \lambda)$ for two distinct values of K . We can see that for each K the dependence on λ and N is similar and resembles $\mathcal{R}_+(\zeta'_0|N, \lambda)$. For contraction with other fields (χ_K and from $GSO(-)$ $\xi'_0, \vartheta_K, \eta_K$) the results are similar, so we omit them.

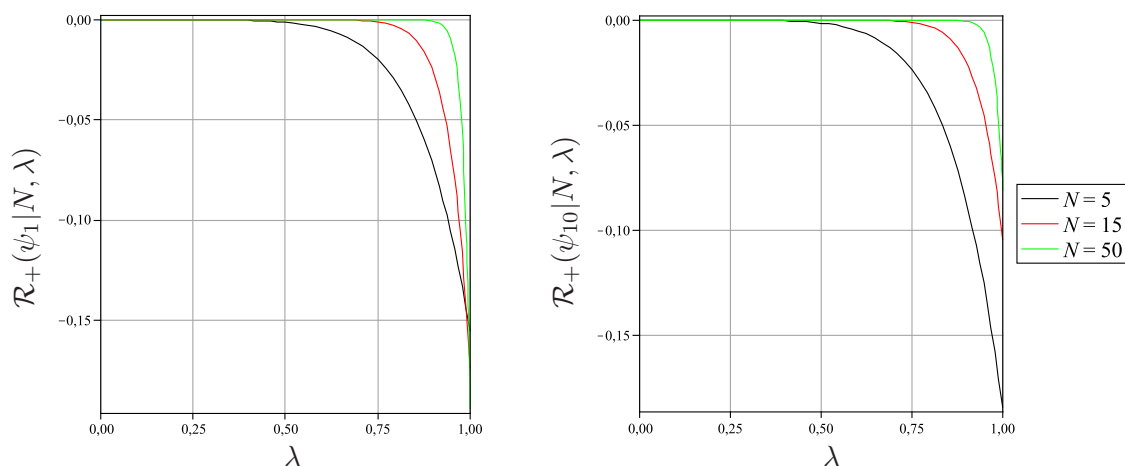


Figure 2. Contraction of equations of motion for $\Phi_{+,N}(\lambda)$, $N = 5, 10, 15, 20, 50$ with ψ_1 and ψ_{10} .

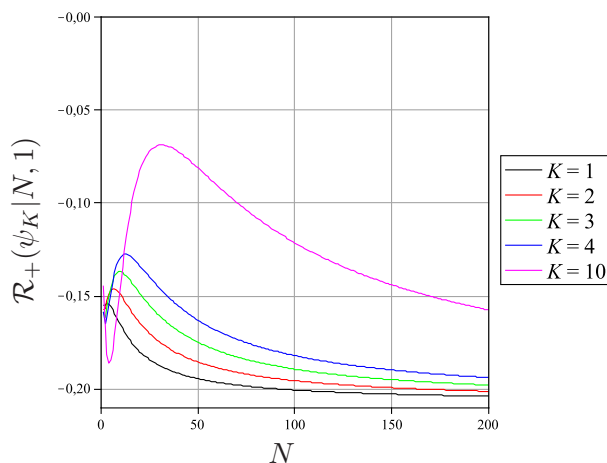


Figure 3. Contraction of equations of motion for $\Phi_{+,N}(1)$ with ψ_K for $K = 1, 2, 3, 4, 10$.

Let us study more thoroughly equations of motion for pure gauge configuration at $\lambda = 1$. In figure 3 we can see that $\mathcal{R}_+(\psi_K|N, 1)$ does not go to zero with $N \rightarrow \infty$. We can make a claim that for different values of $K \lim_{N \rightarrow \infty} \mathcal{R}_+(\psi_K|N, 1)$ is the same. To support this claim we have calculated this expression at $N = 1000$:

K	1	2	10
$\mathcal{R}_+(\psi_K 1000, 1)$	-0.20649	-0.20594	-0.19605

Thus we claim that the limit is about -0.2 and the equations of motion do not hold. This means that the pure gauge configuration $\Phi_{S,N}(\lambda)$ is not an asymptotical solution at $\lambda = 1$.

4.2 Phantom terms and the equation of motion on some low states

We have seen that pure gauge configuration doesn't solve equations of motion at $\lambda = 1$, we need the phantom terms. In this subsection we will show that both phantom terms with

N	a	b	c	d
1	-1.009276758	-0.4947687297	-1.021127911	-0.4547206603
5	-1.002887587	-0.4802195028	-0.9958034182	-0.5258588923
10	-1.001361647	-0.4839491795	-0.9996956796	-0.5033484993
50	-0.999973303	-0.5013822672	-0.9999993154	-0.5000348047
100	-0.999996393	-0.5003673241	-0.9999999396	-0.5000061487

Table 1. Numerical evaluation of coefficients in phantom terms in AGM solution.

exactly the coefficients written above are necessary to satisfy equations of motion. First, we rewrite the partial solution with arbitrary coefficients

$$\begin{aligned}\Phi_{+,N}^R(a,b) &= \sum_{n=0}^N \zeta'_n + a\zeta_N + b\zeta'_N, \\ \Phi_{-,N}^R(c,d) &= \sum_{n=0}^N \xi'_n + c\xi_N + d\xi'_N.\end{aligned}\tag{4.1}$$

To determine a and b coefficients it's enough to contract equations of motion for $\Phi_{+,N}^R$ with ζ'_0 and ψ_1

$$\begin{cases} \mathcal{R}_+(\zeta'_0|N, 1, a, b) = 0, \\ \mathcal{R}_+(\psi_1|N, 1, a, b) = 0. \end{cases}\tag{4.2}$$

Here

$$\mathcal{R}_+(\text{field}|N, 1, a, b) = \langle\langle \text{field}, Q\Phi_{+,N}^R + \Phi_{+,N}^R \star \Phi_{+,N}^R - \Phi_{-,N}^R \star \Phi_{-,N}^R \rangle\rangle.\tag{4.3}$$

Similarly, c and d coefficients can be obtained by contracting equations of motion for $\Phi_{-,N}^R$ with ξ'_0 and ϑ_1

$$\begin{cases} \mathcal{R}_-(\xi'_0|N, 1, c, d) = 0, \\ \mathcal{R}_-(\vartheta_1|N, 1, c, d) = 0. \end{cases}\tag{4.4}$$

Here

$$\mathcal{R}_-(\text{field}|N, 1, a, b) = \langle\langle \text{field}, Q\Phi_{-,N}^R + \Phi_{+,N}^R \star \Phi_{-,N}^R - \Phi_{-,N}^R \star \Phi_{+,N}^R \rangle\rangle.\tag{4.5}$$

By solving these equations numerically for several values of N we obtain the results shown in table 1. From this table we see that these values agree with exact values $a = c = -1$ and $b = d = -\frac{1}{2}$ with good precision.

4.3 Phantom terms and equations of motion on higher states

As we have seen in the previous subsection at $\lambda = 1$ to have a solution of equations of motion in weak sense on the four states (4.2), (4.4) we have to add to the pure gauge solution two phantom terms

$$\begin{aligned}\Phi_{+,N}^R &= \sum_{n=0}^N \zeta'_n - \zeta_N - \frac{1}{2}\zeta'_N, \\ \Phi_{-,N}^R &= \sum_{n=0}^N \xi'_n - \xi_N - \frac{1}{2}\xi'_N.\end{aligned}\tag{4.6}$$

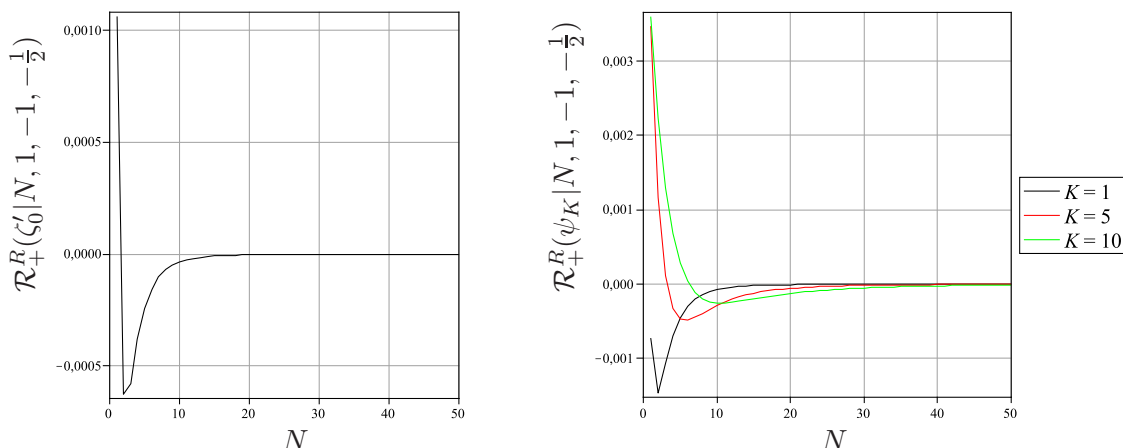


Figure 4. Contraction of equations of motion for $\Phi_{+,N}^R$ with ζ'_0 and ψ_K for $K = 1, 5, 10$.

N	$2\pi^2 S(\Phi_{+,N}, \Phi_{-,N})$
1	0.998538383
5	0.999990263
10	0.999999488
15	0.999999919
20	0.999999979
30	0.999999997

Table 2. Numerical evaluation of action on partial AGM solutions.

In this subsection we check that these terms also provide the validity of equations of motion on higher states. For this purpose we consider the following correlators

$$\mathcal{R}_+^R \left(\text{field}|N, 1, -1, -\frac{1}{2} \right), \quad R_-^R \left(\text{field}|N, 1, -1, -\frac{1}{2} \right), \quad (4.7)$$

where field is one of $\zeta'_0, \psi_K, \chi_K, \xi'_0, \vartheta_K, \eta_K$

In figure 4 we see $\mathcal{R}_+^R(\zeta'_0|N, 1, -1, -\frac{1}{2})$ and $\mathcal{R}_+^R(\psi_K|N, 1, -1, -\frac{1}{2})$ for three values of K . We can see that for large N $\Phi_{+,N}^R$ asymptotically solves the equation of motion. We also see that $\mathcal{R}_+^R(\psi_K|N, 1, -1, -\frac{1}{2})$ as a function of N has an extremum near $N = K$. We can think that the largest contribution comes from correlators where $N \approx K$. This means that $\mathcal{R}_+^R(\psi_K|N, 1, -1, -\frac{1}{2})$ must be small for $N \gg K$ because $\Phi_{+,\infty}^R$ is a solution. For other fields we obtained the same results.

4.4 Calculation of action

Let us calculate the action on the AGM solution. We calculate the action on partial sums and present the result depending on index N of partial sum. The action is expected to be $\frac{1}{2\pi^2}$, so we multiply it by $2\pi^2$ to compare with 1. Numerical results are presented in the

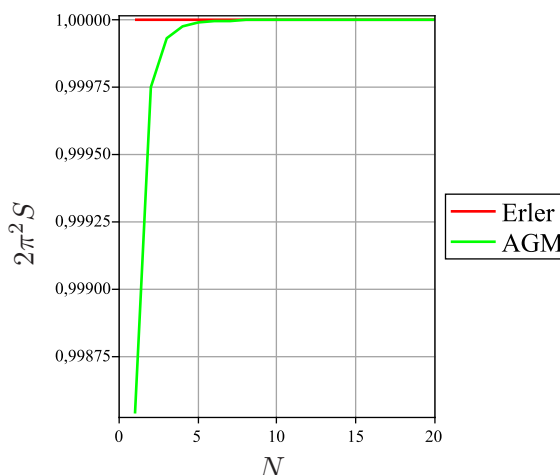


Figure 5. Dependence of action on N .

table 1 and in figure 5. We see that action has a good convergence to the expected value. This proves the first Sen conjecture.

For comparison on the same figure we have presented the action on the Erler solution. We can see that it doesn't depend on N . We also see that both curves join asymptotically. This figure forces us to conclude that $GSO(-)$ does not contribute to the value of action and it provides a numerical confirmation of the claim given in [30].

5 Pure gauge configurations and the Schnabl solution

5.1 Contractions for pure gauge configurations

Let us for completeness consider the Schnabl pure gauge configuration (2.31) that has been already done in many details [1, 3, 4]. To check by direct computations whether (2.43) solves equations of motion in weak sense on the subspace spanned by φ_K we evaluate expression (2.42). The results of calculations are presented in figure 6. Here $\mathcal{R}_B(\varphi_1|N, \lambda)$ is shown as a function of N and λ . We see that for the large N and $\lambda < 1$ the surface is closer to the (λ, N) -plane meanwhile when $\lambda > 1$ the surface blow up. A similar picture one gets for any K (see [34] for more details). For $\lambda = 1$ we observe numerically an interesting phenomena that for any N there is a specific $K = K_{\max}(N)$ so that $|\mathcal{R}_B(\varphi_{K_{\max}(N)}|N, 1)| > R_0$, where R_0 is an universal constant and one can see that $R_0 > 0.26$ (see figure 6.B).

This means that the perturbative pure gauge configuration does not solve the equation of motion in the uniformly weak sense on a subspace spanned by φ_K for $\lambda = 1$ but does solve it in the weak sense. A similar picture we get for contractions with φ'_K . This fact explains the observation [3, 4] that the perturbative pure gauge configuration does not solve the equation of motion contracted with this configuration itself. We can also demonstrate this numerically.

For this purpose we evaluate the correlators

$$\mathcal{R}_B(\Phi_{B,K}|N, \lambda) \equiv \langle \Phi_{B,K}(\lambda), Q\Phi_{B,N}(\lambda) + \Phi_{B,N}(\lambda) \star \Phi_{B,N}(\lambda) \rangle, \quad (5.1)$$

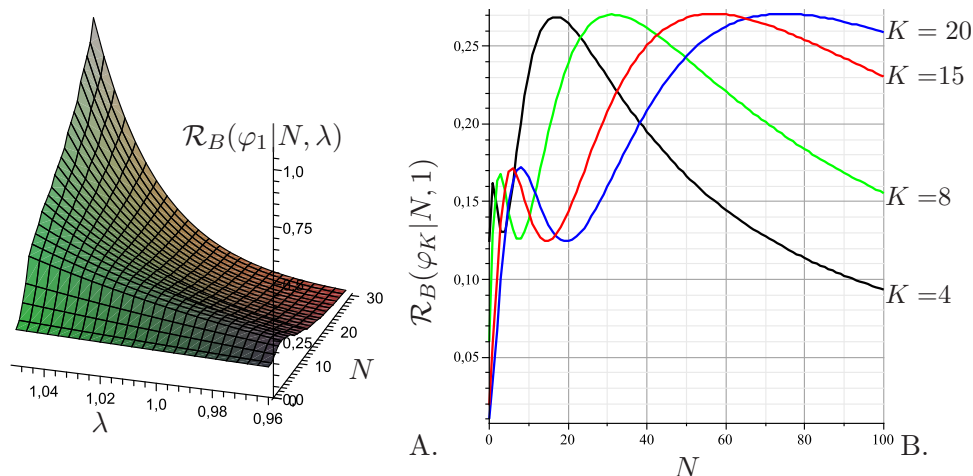


Figure 6.
A. Contraction of equation of motion for $\Phi_{B,N}(\lambda)$ with φ_1 , $N \leq 30$, $0.96 < \lambda < 1.06$,
B. Contraction of the equation of motion for $\Phi_{B,N}(\lambda)$ with φ_K , $1 \leq N \leq 100$, $\lambda = 1$ and $K = 4, 8, 15, 20$.

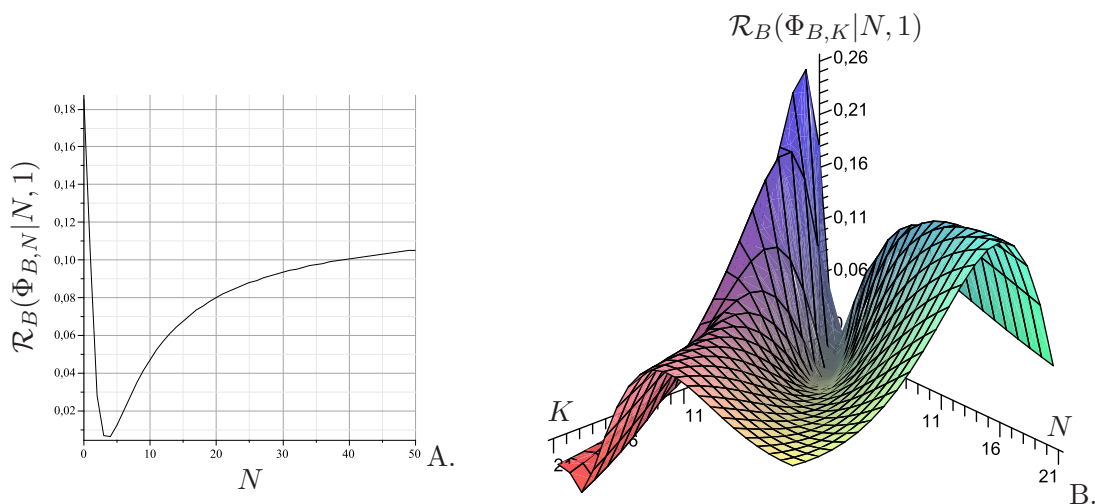


Figure 7. Contraction of the equation of motion for $\Phi_{B,N}(1)$ with $\Phi_{B,K}(1)$:
A. $N = K$ and $1 \leq N \leq 50$;
B. $1 \leq N \leq 20$, $1 \leq K \leq 20$.

and draw $\mathcal{R}_B(\Phi_{B,K}|N, \lambda)$ for $K = N$ as a function of N for $\lambda = 1$. In figure 7.A we see that $\mathcal{R}_B(\Phi_{B,N}|N, 1)$ does not go to zero when $N \rightarrow \infty$.

In figure 7.B we show the dependence of $\mathcal{R}_B(\Phi_{B,K}|N, 1)$ on K and N and one can see that there are directions along which $\mathcal{R}_B(\Phi_{B,K}|N, 1)$ does not go to zero when $N, K \rightarrow \infty$.

5.2 Contractions for the Schnabl solution

Since the pure gauge configuration doesn't solve equation of motion at $\lambda = 1$ one can try to add the phantom term. In this subsection we will show numerically that this phantom term minimize deviations from the solution. We add the Schnabl phantom term to the

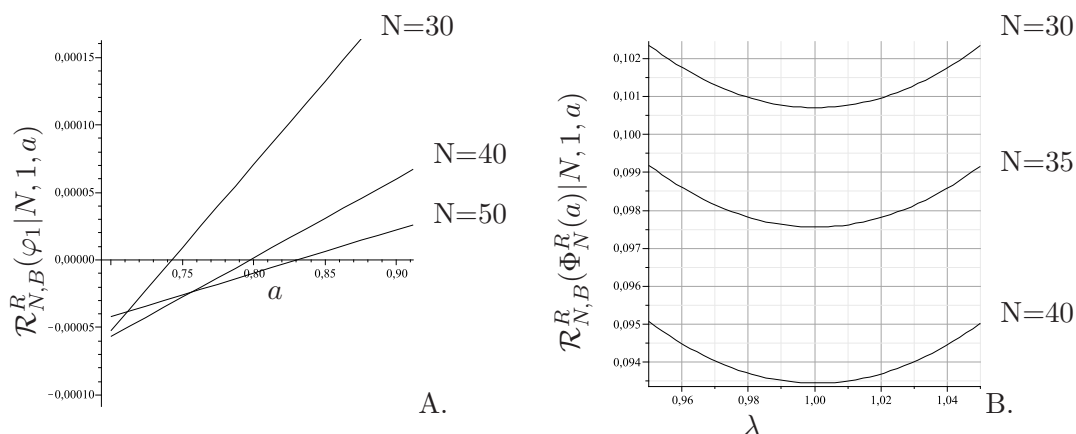


Figure 8.

A. Contraction of the equation of motion for $\Phi_{B,N}^R(a)$ with φ_1 , as function of a for different values of N , $N = 30, 40, 50$.

B. $\mathcal{R}_{N,B}^R(\Phi_N^R(a)|N, 1, a)$ as a function of different values of N , $N = 30, 35, 40$.

pure gauge solution with an arbitrary coefficient⁶

$$\Phi_{B,N}^R(a) = -\Phi_{B,N}(1) + a\varphi_N \tag{5.2}$$

and consider

$$\mathcal{R}_B^R(\varphi_K|N, 1, a) = \langle \varphi_K, Q\Phi_{B,N}^R(a) + \Phi_{B,N}^R(a) \star \Phi_{B,N}^R(a) \rangle. \tag{5.3}$$

In figure 8.A we plot $\mathcal{R}_B^R(\varphi_1|N, 1, a)$ for different values of a and $N = 30, 40, 50$. We see that $\mathcal{R}_B^R(\varphi_1|N, 1, a)$ is equal to zero for these particular value of N for $a = a(N)$, which is very closed to 1 and a deviation from 1 becomes smaller when N increases.

In figure 8.B we plot $\mathcal{R}_B^R(\Phi_N^R(a)|N, 1, a)$ for different values of a and N . We see that $a = 1$ minimize the deviation from the zero of $\mathcal{R}_B^R(\Phi_N^R(a)|N, 1, a)$ for particular values of N and this deviation decreases when N increases.

5.3 Calculation of action on the Schnabl solution

It is known that the value of the action on the Schnabl solution multiplied on $2\pi^2$ is equal to -1. This has been checked in [1, 3, 4] in the sense that

$$S = \lim_{N \rightarrow \infty} S(N) = -\frac{1}{2\pi^2}, \tag{5.4}$$

where we use the following notations

$$S(N) \equiv S(\Phi_{B,N}^R(1)). \tag{5.5}$$

⁶We perform calculations in [1, 4] notations. To fit our general notations in section 2 we put the minus in front of the first term of (5.2).

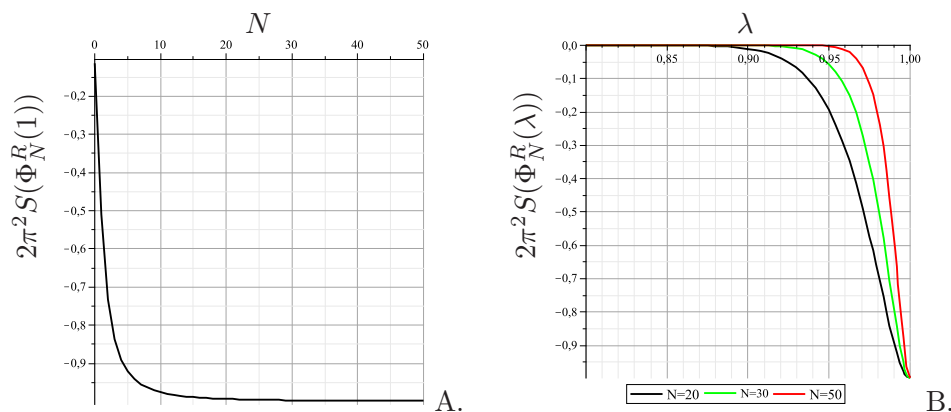


Figure 9. A. Value of the action multiplied by $2\pi^2$ on $\Phi_N^R(\lambda, 1)$ as function of N for $\lambda = 1$; B. Value of the action multiplied by $2\pi^2$ on $\Phi_N^R(\lambda, 1)$ as function of λ for $N = 20, 30, 50$.

We calculate the action on partial sums (5.5) for $a = 1$ and present the graph for $S(N)$ in figure 9.A. We also calculate the action on $\Phi_N^R(\lambda, 1) = -\Phi_{B,N}(\lambda) + \lambda^{N+1}\varphi_N$ and plot $\Phi_N^R(\lambda, 1)$ as function of λ for $N = 20, 30, 50$ in figure 9.B. We see that the values of $2\pi^2\Phi_N^R(\lambda, 1)$ are closed to -1 when $\lambda \rightarrow 1$ and N is big enough.

6 Conclusion and outlook

In this paper we have studied the special class of pure gauge configurations in the Witten bosonic SFT, the cubic SSFT and the fermionic SFT including the $GSO(-)$ sector. All these configurations are parameterized by one parameter λ and are constructed as perturbation expansions in λ . One can expect that these configurations solve corresponding string field equations of motion. However since they are constructed as perturbation expansions the best that we can expect is the validity of the corresponding perturbative expansions at each order of the perturbation parameter. To find physical quantities related to these configurations one has to deal with these configurations as a whole. Therefore one needs to study the convergence of the perturbation expansions.

The simplest possibility to deal with the convergence problem is to consider existence of weak asymptotic solutions to equations of motion in a sense of definition (2.35). Already in this simplest framework we have seen that for the large parameter of the perturbation expansion the pure gauge truncated configurations give divergent contributions to the equation of motion on the subspace of the wedge states. In particular, on the example of the Erler pure gauge configuration in the SSFT one can see explicitly (3.2) that the perturbative pure gauge configuration does not solve the equation of motion at $\lambda \geq 1$. We have also seen numerically the similar effect for the pure gauge configurations related to tachyon solutions for the bosonic and the NS fermionic SFT. We have seen a difference in the behavior of correlators of the equations of motion for string fields with wedge states for the bosonic and fermionic cases. For the bosonic case the equation of motion is not

satisfied in the *uniform* weak sense while for the fermionic cases the equation of motion is not satisfied already in the weak sense at $\lambda = 1$.

By analytical calculations for the SSFT case and by the numerical ones for the tachyon cases we have shown that the perturbation expansions are cured by adding extra terms. In the tachyon cases these terms coincide with the terms found before in [1] and [30] for the bosonic and fermionic string, respectively, from requirements to implement the Sen conjectures. In the case of superstring these extra terms also coincide with the Erler phantom terms and this justified the Erler choice of these terms since a priori there is no reason to have a special number for the value of the action on this solution.

All currently known analytical solutions can be cast in a form of formal gauge solutions and one can hope that all string field theory solutions are of this form (see detailed discussion of this issue in the Fuchs and Kroyter recent review [32]). A check of the equation of motion in a weak sense on wedge states could help to find a simple prescription for regularizing formal solutions.

In particular, it is worth to study this problem for time depending rolling tachyon solutions⁷ that have been constructed perturbatively [7, 8, 17, 18, 27, 28]. As has been found by Ellwood [14] the late time behavior of the rolling tachyon solution [7, 8] approaches Schnabl's solution in the sense of correlators with fields belonging to the Fock space. However the phantom term does not show up in this consideration.

As has been noted in [29] a similar limit for the superstring [27, 28] fails to yield a well-defined expression and it would be interesting to study a similar problem for the solution with nonvanishing $GSO(-)$ sector.

All pure gauge configurations are gauge equivalent, but the singularity problem and the necessity of adding the extra terms means that the gauge equivalence can be violated for non-trivial solutions and this question needs of a rather delicate study. It may turn out that it is not enough to calculate only the action to study gauge equivalence, so we have to consider other gauge invariants. Full list of gauge invariant quantities is unknown, but it includes the invariants related to the 1-point disk scattering amplitudes of closed strings [55, 56]. It would be interesting to clarify the question about gauge equivalence of the Erler and AGM solutions performing 1-point disk calculations.

Rolling tachyon solutions [58, 61] play important role in cosmology [66–72]. These solutions exist in the flat background within the level truncation scheme for the fermionic string [61, 62, 65], however for the bosonic string wide oscillations do exist. Moreover, there is the no-go theorem about existence of rolling solutions for a toy model of the bosonic level truncated model, that is the p -adic string model for $p = 2$ [57]. There are existence theorems for $p = 3$ p -adic string model [63] and similar models [64], which are the toy models of the fermionic level truncated model with $GSO(-)$ sector. Note that a nontrivial background can change the situation for toy models as well as for realistic models [70]. The wide oscillation behavior is in the apparent conflict with the exponential grow result found using BSFT and a resolution of this contradiction can be associated with the non-local field redefinition between the two theories [59]. A direct evaluation of the partition

⁷For early constructions of exact solutions in open bosonic string field theory using marginal deformation in CFT [50] see [51–54]

function of the rolling tachyon solution of [7, 8], gives a result very similar to the one obtained in BSFT [60]. From cosmological perspectives results [70] could mean that the field redefinition becomes more smooth in the FRW background. Note also cosmological applications [71, 72] of the Hellerman and Schnabl light-like rolling solutions in SFT [73].

Acknowledgments

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A Split-strings formalism

Our split-string notations are based on Okawa's paper [3]. A state written in the split-string notation can be rewritten in the conformal language. For example:

$$\begin{aligned}
 Fc\Omega^n BcF &= c(0)|0\rangle \star |n\rangle \star B_1^L c(0)|0\rangle \\
 &= \frac{1}{\pi} U_{n+2}^\dagger U_{n+2} \left[(\mathcal{B}_0 + \mathcal{B}_0^\dagger) \tilde{c}\left(\frac{\pi}{4}n\right) \tilde{c}\left(-\frac{\pi}{4}n\right) - \frac{\pi}{2} \left(\tilde{c}\left(\frac{\pi}{4}n\right) + \tilde{c}\left(-\frac{\pi}{4}n\right) \right) \right] |0\rangle.
 \end{aligned}
 \tag{A.1}$$

One uses split-string notations to calculate correlators on a cylinder of some circumference and these correlators can be reduced to the correlators on the unit disc by a suitable conformal transformation. This provides the connection between the split-string formalism and the familiar conformal language.

Further we collect some notations and useful formulae. We use following string fields

$K = K_1^I I\rangle$	Grassmann even, gh	$\# = 0,$
$B = B_1^I I\rangle$	Grassmann odd, gh	$\# = -1,$
$c = c(0) I\rangle$	Grassmann odd, gh	$\# = 1,$
$\gamma = \gamma(0) I\rangle$	Grassmann even, gh	$\# = 1,$
$\gamma^2 = \gamma^2(0) I\rangle$	Grassmann even, gh	$\# = 2,$

(A.2)

which satisfy the following algebraic relations:

$$\begin{aligned}
 \{B, c\} &= 1, & [K, B] &= 0, & B^2 &= c^2 = 0, \\
 [B, \gamma] &= 0, & & & [c, \gamma] &= 0, \\
 dK &= 0, & & & dB &= K, \\
 dc &= cKc - \gamma^2, & & & & \\
 d\gamma &= cK\gamma - \frac{1}{2}\gamma Kc - \frac{1}{2}\gamma cK, & & & & \\
 d\gamma^2 &= cK\gamma^2 - \gamma^2 Kc, & & & &
 \end{aligned}
 \tag{A.3}$$

where $d = Q_B$ is the BRST operator.

We also use

$$F = e^{\frac{\pi}{4}K} = \Omega^{\frac{1}{2}}, \tag{A.4}$$

which is the square root of the $SL(2, \mathbb{R})$ vacuum $\Omega = e^{\frac{\pi}{2}K}$

B Correlators

Using the split-string formalism we obtain the following correlators

B.1 Quadratic correlators

$$\begin{aligned}
 \langle\langle \psi_n, Q\psi_k \rangle\rangle &= \frac{n+k+2}{\pi^2}, \\
 \langle\langle \chi_n, Q\chi_k \rangle\rangle &= 0, \\
 \langle\langle \psi_n, Q\chi_k \rangle\rangle &= \langle\langle \chi_k, Q\psi_n \rangle\rangle \\
 &= -\frac{1}{\pi^3} \left[(k+2) \cos\left(\frac{\pi k}{n+k+2}\right) - \frac{\pi nk}{n+k+2} \sin\left(\frac{\pi k}{n+k+2}\right) \right], \\
 \langle\langle \vartheta_n, Q\vartheta_k \rangle\rangle &= \langle\langle \eta_m, Q\eta_k \rangle\rangle \\
 &= -\frac{1}{\pi^3} \left[\frac{k-n}{2} \cos\left(\frac{\pi(n+1)}{n+k+2}\right) + \frac{\pi(nk-1)}{n+k+2} \sin\left(\frac{\pi(n+1)}{n+k+2}\right) \right], \\
 \langle\langle \vartheta_n, Q\eta_k \rangle\rangle &= \langle\langle \vartheta_k, Q\eta_n \rangle\rangle = \langle\langle \eta_m, Q\vartheta_k \rangle\rangle = \langle\langle \eta_k, Q\vartheta_n \rangle\rangle \\
 &= -\frac{1}{\pi^3} \left[\cos\left(\frac{\pi}{n+k+2}\right) - \frac{\pi(n+k+1)}{n+k+2} \sin\left(\frac{\pi}{n+k+2}\right) \right]. \tag{B.1}
 \end{aligned}$$

B.2 Cubic correlators

Correlators without 0-th term:

$$\begin{aligned}
 \langle\langle \psi_n, \psi_m, \chi_k \rangle\rangle &= -\frac{n+m+k+3}{\pi^3} \cos\left(\frac{\pi k}{n+m+k+3}\right), \\
 \langle\langle \psi_n, \vartheta_m, \vartheta_k \rangle\rangle &= -\frac{n+m+k+3}{\pi^3} \cos\left(\frac{\pi(m+1)}{n+m+k+3}\right), \\
 \langle\langle \psi_n, \vartheta_m, \eta_k \rangle\rangle &= \frac{n+m+k+3}{\pi^3} \cos\left(\frac{\pi(n+2)}{n+m+k+3}\right), \\
 \langle\langle \psi_n, \eta_m, \eta_k \rangle\rangle &= -\frac{n+m+k+3}{\pi^3} \cos\left(\frac{\pi(k+1)}{n+m+k+3}\right). \tag{B.2}
 \end{aligned}$$

Correlators with one 0-th term:

$$\begin{aligned}
 \langle\langle \zeta'_0, \psi_n, \psi_k \rangle\rangle &= -\frac{n+k+3}{2\pi^2}, \\
 \langle\langle \zeta'_0, \psi_n, \chi_k \rangle\rangle &= \langle\langle \zeta'_0, \chi_k, \psi_n \rangle\rangle = -\frac{1}{\pi^3} \left[\cos\left(\frac{\pi k}{n+k+3}\right) + \frac{\pi k}{n+k+3} \sin\left(\frac{\pi k}{n+k+3}\right) \right], \\
 \langle\langle \zeta'_0, \chi_n, \chi_k \rangle\rangle &= 0, \\
 \langle\langle \xi'_0, \psi_n, \eta_k \rangle\rangle &= -\langle\langle \xi'_0, \vartheta_n, \psi_k \rangle\rangle \\
 &= \frac{1}{\pi^3} \left[\cos\left(\frac{\pi}{n+k+3}\right) - \frac{\pi(n+k+2)}{n+k+3} \sin\left(\frac{\pi}{n+k+3}\right) \right], \\
 \langle\langle \xi'_0, \psi_n, \vartheta_k \rangle\rangle &= -\langle\langle \xi'_0, \eta_k, \psi_n \rangle\rangle \\
 &= -\frac{1}{\pi^3} \left[\frac{n+k+7}{2} \cos\left(\frac{\pi(n+2)}{n+k+3}\right) + \frac{\pi(n-k+1)}{n+k+3} \sin\left(\frac{\pi(n+2)}{n+k+3}\right) \right], \\
 \langle\langle \zeta'_0, \vartheta_n, \vartheta_k \rangle\rangle &= -\frac{1}{\pi^3} \left[\cos\left(\frac{\pi(n+1)}{n+k+3}\right) + \frac{\pi(n+1)}{n+k+3} \sin\left(\frac{\pi(n+1)}{n+k+3}\right) \right],
 \end{aligned}$$

$$\begin{aligned}
\langle\langle \zeta'_0, \vartheta_n, \eta_k \rangle\rangle &= \frac{1}{\pi^3} \left[\cos\left(\frac{2\pi}{n+k+3}\right) - \frac{\pi(n+k+1)}{n+k+3} \sin\left(\frac{2\pi}{n+k+3}\right) \right], \\
\langle\langle \zeta'_0, \eta_m, \vartheta_k \rangle\rangle &= 0, \\
\langle\langle \zeta'_0, \eta_m, \eta_k \rangle\rangle &= -\frac{1}{\pi^3} \left[\cos\left(\frac{\pi(k+1)}{n+k+3}\right) + \frac{\pi(k+1)}{n+k+3} \sin\left(\frac{\pi(k+1)}{n+k+3}\right) \right]. \tag{B.3}
\end{aligned}$$

Correlators with two 0-th terms:

$$\begin{aligned}
\langle\langle \zeta'_0, \zeta'_0, \psi_n \rangle\rangle &= -\frac{1}{\pi^2}, \\
\langle\langle \zeta'_0, \zeta'_0, \chi_n \rangle\rangle &= \frac{n^2}{\pi(n+3)^3} \cos\left(\frac{\pi n}{n+3}\right), \\
\langle\langle \zeta'_0, \xi'_0, \vartheta_n \rangle\rangle &= -\langle\langle \xi'_0, \zeta'_0, \eta_n \rangle\rangle = -\frac{n+2}{\pi(n+3)^3} \cos\left(\frac{\pi}{n+3}\right), \\
\langle\langle \zeta'_0, \xi'_0, \eta_n \rangle\rangle &= +\frac{1}{\pi^3} \left[\left(\frac{1}{2} + \frac{\pi^2(1-n^2)}{(n+3)^3}\right) \cos\left(\frac{2\pi}{n+3}\right) - \frac{\pi(n+1)}{2(n+3)} \sin\left(\frac{2\pi}{n+3}\right) \right], \\
\langle\langle \xi'_0, \zeta'_0, \vartheta_n \rangle\rangle &= -\frac{1}{\pi^3} \left[\left(\frac{1}{2} + \frac{\pi^2(1-n^2)}{(n+3)^3}\right) \cos\left(\frac{2\pi}{n+3}\right) - \frac{\pi(n+1)}{2(n+3)} \sin\left(\frac{2\pi}{n+3}\right) \right], \\
\langle\langle \xi'_0, \xi'_0, \psi_n \rangle\rangle &= \frac{1}{\pi^3} \left[\left(-1 + \frac{\pi^2 n(n+2)}{(n+3)^3}\right) \cos\left(\frac{\pi}{n+3}\right) + \frac{\pi(n+2)}{n+3} \sin\left(\frac{\pi}{n+3}\right) \right]. \tag{B.4}
\end{aligned}$$

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